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# Corrections to A Meticulous Introduction to Stochastic Calculus



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Lines are counted outside the grey boxes.

**2016, last update: June 30, 2017**

We thank Andreas Basse-O'Connor and Andreas Kruse Møller for a lot of the 299 corrections below, which amounts to approximately 0.943 corrections to be made per page.

**Page i, line 11** Replace “Explicit” with “Explicitly”.

**Page ii, line 11 from the top:** “well-know” should be “well-known”.

**Page ii, line 14 from the top:** “Integrals” should be “integral”.

**Page ii, line 15 from the top:** “integration of stochastic processes” should be “integration with respect to stochastic processes”.

**Page ii, line 15 from bottom:** “Discuss” is spelled wrong.

**Page 1, line 3 below Definition 1.1:** “are” should be replaced by “is”.

**Page 1, second line in Lemma 1.2:** There’s a “the” missing between “Then” and “following”.

**Page 2, line 11 from the bottom:** Strict inequality should be replaced by weak inequality at  $M + \varepsilon/2$ .

**Page 3, Theorem 1.5:** polish should be with capital P.

**Page 3, line 2 in the proof of (i):** “find a  $N$ ” should be replaced by “find an  $N$ ”.

**Page 3, first line in the proof of (iii):** “there is a  $s \in [\dots]$ ” should be replaced by “there is an  $s \in [\dots]$ ”.

**Page 4, proof of Theorem 1.5(iv):**  $F$  should probably be specified as closed. Moreover there are at least four  $\sigma_A$  that should be  $\sigma_F$ .

**Page 4, line 6 in Definition 1.7:** there is a “be” missing. It should be “Let  $(Y_t)$  be a [...]”.

**Page 4, proof of Lemma 1.6(ii):** There’s parenthesis missing, so the calculation is wrong. It should be something like:

$$\{\inf_{n \in \mathbb{N}} T_n < t\} = \{\inf_{n \in \mathbb{N}} T_n \geq t\}^c = \left( \bigcap_{n \in \mathbb{N}} \{T_n \geq t\} \right)^c = \bigcup_{n \in \mathbb{N}} (\{T_n \geq t\}^c) = \bigcup_{n \in \mathbb{N}} \{T_n < t\} \in \mathcal{F}_t.$$

**Page 5, middle of the page:** In the definition of  $Y_\tau$ , the limit should depend on  $\omega$ , i.e.  $\lim_{n \rightarrow \infty} Y_n(\omega)$ . Moreover “as.” should be replaced by “as”.

**Page 5, Theorem 1.9(ii):** Maybe writing  $(\mathcal{F}_t)$ -adapted is a bit more clear.

**Page 5, Theorem 1.9(v):** Delete “the”.

**Page 6, line 4 in (iii):** There are parenthesis missing. It should be:

$$([0, \ell] \times \Omega) \cup ([0, t] \times \{\tau < \ell\}).$$

**Page 6:** The last measurability in item (iii) obviously doesn’t make sense. It should be:  $(\mathcal{B}([0, t]) \otimes \mathcal{F}_t)$ - $\mathcal{B}(S)$ -measurable; corresponding to Definition 1.7.

**Page 7, line 10 from the bottom:** “then  $\tau_n(\omega)$ ” should be replaced by “ $\tau_n(\omega)$ ”.

**Page 9, line 7 from the bottom:** “Note that we also the identity” should be replaced by “Note that we also have the identity”.

**Page 9, line 6 from bottom:** There should be a “the” in between “to” and “reader”.

**Page 10, line 4 below Remark 1.14:** The statement “and  $\tau$  is a stopping time” should be removed.

**Page 10, Theorem 1.15 part (iii):** We still assume that  $H_t$  is predictable.

**Page 11, Lemma 1.16:** Should not be there. Its a leftover lemma from an alpha version of the book.

**Page 11, proof of Lemma 1.18:** “null= $\beta$ sequence” should be “null-sequence”.

**Page 12, right after Remark 1.19:** The definition should be written  $\|f\|_N = \sup_{t \leq N} |f(t)|$ .

**Page 12, Lemma 1.20:** The line after statement (i) and (ii) should be: “Then (i) implies (ii) and (ii) implies [...]”.

**Page 13, around middle:**  $Q_m \in \mathcal{F}_0$  should be  $Q_n \in \mathcal{F}_m$

**Page 13, line 14 from top:** “an  $K$ ” should be replaced by “a  $K$ ”.

**Page 13, second to last equation:**

$$X_t^{nk}(\omega) = X_t^{n1}(\omega) + \sum_{i=1}^{k-1} X_t^{ni+1}(\omega) - X_t^{ni}(\omega).$$

**Page 14, line 4 in the proof:** “and” should be deleted.

**Page 17, Exercise 1.3:** Don’t use the hint. Use 1.9 and 1.15.

**Page 18, line 1 from top:** “denotes Euclidean” should be replaced by “denotes the Euclidean”.

**Page 19, line 4 from bottom:** Replace “shows” with “show”.

**Page 21, Definition 2.6(iv):** Replace “a  $(\mathcal{F}_t)_{t \in T}$ ” with “an  $(\mathcal{F}_t)_{t \in T}$ ”. Item (iv) should not be a definition for itself but rather be under item (iii).

- Page 21, Remark 2.7(D):** Should be "an  $\mathcal{F}_t$ ".
- Page 21, Remark 2.7(E):** "an martingale" should be "a martingale".
- Page 31, line after Definition 2.19:** Remove "cf.".
- Page 33, Definition 2.27:** The statement should be "Let  $(\mathcal{F}_n)$  be a filtration and  $(X_n)$  an  $(\mathcal{F}_n)$ -adapted process. Then an  $(\mathcal{F}_n)$ -stopping time  $\tau$  is said to **optional** for  $(X_n)$  if  $\{X_{n \wedge \tau} \mid n \in \mathbb{N}_0\}$  is uniformly integrable".
- Page 35, Equation (2.5):** Should end with "." instead of ",".
- Page 37, Corollary 2.34:** Accent missing, replace "Levy's Backward Theorem" with "Lévy's Backward Theorem".
- Page 37, Corollary 2.34:** The limit is as  $n$  tends to  $\infty$ .
- Page 40, Lemma 2.38:** We need to assume that  $(\mathcal{F}_t)$  satisfies the Natural Conditions. Otherwise the limiting process will not be adapted.
- Page 40, Lemma 2.38:** Two times of: "a" should be "an".
- Page 42, Proof of Corollary 2.42(i)  $\implies$  (ii):**  $\mathbb{E}[|X_t - X_\infty|] \leq 1$ .
- Page 42, Corollary 2.42 (ii)  $\implies$  (i):** Should be "The Martingale convergence Theorem implies that".
- Page 42, Theorem 2.43:** "Let  $a < b$  be real" should be "Let  $a < b$  be real numbers".
- Page 44-45, Theorem 2.47:** Càdlàg should be defined.
- Page 45, second line (in proof of Theorem 2.47):** Wrong accents on "càdlàg".
- Page 46, Lemma 2.49, third line** "times" should be "time".
- Page 47, Theorem 2.50:** Last word should be "times" not "time".
- Page 52, Exercise 2.2, last line:** "a" should be "an".
- Page 54, Lemma 3.2(ix):** One should have " $\leq$ " instead of " $=$ ".
- Page 54, Lemma 3.2(ii):** In the inequality  $\lambda$  should be replaced by  $\alpha$ .
- Page 54, proof of Lemma 3.2(i) and (ii):** Both calculations should end with "." instead of ",".
- Page 59, line 6 from the top:** Replace "value" with "values".
- Page 59, Lemma 3.3, first line:** It should say "if and only if".
- Page 63, Line 8 from top:** A hyphen is missing between Lebesgue and Stieltjes.
- Page 63, line 16 from top:** Missing an "a". It should say  $dh_{2,a}$ .
- Page 67, line 5 from bottom:** Replace "a  $N \in \mathbb{N}$ " with "an  $N \in \mathbb{N}$ ".
- Page 71, Example 3.12:** The third last line starts with a space. Oops.
- Page 80, line 5 from bottom:** Misplaced colon.
- Page 88:** One should interchange the order of the two exercises.
- Page 89:** "Integrands" should be removed from the headline/chapter name.
- Page 89, Definition 4.1:** We should assume that  $(X_t)$  is progressively measurable.
- Page 89, Definition 4.1, line 3:** Replace "a" with "an".
- Page 89, Remark 4.2(B):** In the fifth line "a  $(\mathcal{F}_t)$  martingale" should be "an  $(\mathcal{F}_t)$ -martingale".

**Page 89, Remark 4.2(D):** One should use the word “corresponding” instead of “matching” in line 2.

**Page 90, proof of Theorem 4.3, line 2:** (iii) follows from (i) and (ii).

**Page 90, proof of Theorem 4.3:** Instead of “E.g.” one should use “I.e.”

**Page 91, line 3 in proof of Theorem 4.6:** Replace “is a  $(\overline{\mathcal{F}}_t)$ ” with “is an  $(\overline{\mathcal{F}}_t)$ ”.

**Page 92, Definition 4.8:**  $Y_t - Y_s$  in (iii) should be  $X_t - X_s$ .

**Page 95:** We should assume from now on that  $(\mathcal{F}_t)$  satisfies the natural conditions.

**Page 95, line 4 under Definition 5.1:** “is” should be “are”.

**Page 96, after Equation (5.1):**  $|\Delta_n|$  is the width of the partition defined as  $\sup_{i \in \{1, \dots, n\}} (t_i^n - t_{i-1}^n)$ .

**Page 97:** Last sentence in the first paragraph should be deleted.

**Page 98, line 1 in Step 2:**  $r$  should be strictly larger than 0.

**Page 98, first line in the last equation of Step 2:** Write “ $Q_r^\Gamma(Q^\Delta - Q^{\Delta'})$ ” instead of “ $Q_r^\Gamma(Q_t^\Delta - Q_t^{\Delta'})$ ”.

**Page 98, second line in the last equation of Step 2:** In the last term one should replace “ $t$ ” with “ $r$ ”.

**Page 99, line 1 in the fourth equation:** After the first equality sign one should replace “ $s_{n-1}$ ” with “ $s_n$ ”.

**Page 99, line 2 in the fourth equation:** The sum should start from  $k = 2$  and end at  $k = n + 1$ .

**Page 99, line 6 from the bottom:** Only the first factor goes to 0.

**Page 99, before Step 4:** “go to 0 per” should be replaced with “converges to 0 by”.

**Page 99, last sentence before Step 4:** “Which” should be replaced by “This”.

**Page 99, Step 4:** Remove  $\frac{1}{2}$ , we show the inequality for  $\mathbb{E}[(Q^\Gamma)^2]$ .

**Page 100, third equation from the bottom:** The “,” should be a “.”

**Page 100, first line in Step 6:** The union is not a partition.

**Page 101, line 5 over Lemma 5.5:** “a” should be “an”.

**Page 101, last item, first line:** “and” should be deleted.

**Page 101, last item:**  $t$  is not contained in  $[0, K]$ .

**Page 102, in the middle:** “This then” should be corrected to “this”.

**Page 102, Lemma 5.6 line 1:** Missing “u” in continuous.

**Page 103, Definition 5.8, line 4:** “+” should be “-”.

**Page 103, Definition 5.8, line 2:** Too many “then”s.

**Page 104, second equation:** Partitions of finite intervals are assumed finite.

**Page 104, Theorem 5.11:** in (ii) and (iii) misses the  $t$  on the covariance processes.

**Page 105, line 3 from bottom:** Replace “there exists a  $N$  [...]” with “there exists an  $N$  [...]”.

**Page 107:** One should assume continuity in Exercises 5.5–5.9.

**Page 107, Exercise 5.5:** Here we define  $\langle X \rangle_\infty = \lim_{t \rightarrow \infty} \langle X \rangle_t$ .

**Page 107, Exercise 5.10, first line:** One should write “ $(X_t)$ ” instead of “ $(X)_t$ ”.

- Page 109, first line after second equation:** “Process” should be “Random Variable”.
- Page 109, third line after second equation:** “not necessarily but” should be changed to “usually not”.
- Page 109, fourth line after second equation:** “Buy choosing” should be changed to “By”.
- Page 109, Line 10:** First “is” should be “are”.
- Page 109, Line 12** “But” should be “be”.
- Page 110, proof of Theorem 6.1(d):** Should be changed to “For each  $\omega \in \Omega$  the map  $t \mapsto (H \bullet X)_t(\omega)$  is constant on  $(a, b)^c$ ”.
- Page 111, line 1:** Should be “To a larger class of processes”.
- Page 111, line 6:** “a” should be “an”.
- Page 111, second line after first equation:** “Processes” should be “random variables”.
- Page 113, Theorem 6.3(i):**  $Y$  should be replaced with  $X$ .
- Page 113, Theorem 6.3(ii):** “ $K$ ” should be “ $H$ ”.
- Page 116, line 3:** The reference should be to Lemma 3.23(ii).
- Page 117, Lemma 6.10, first line:** “Process” should be “Random variable”.
- Page 117, proof of Lemma 6.10, line 5:** “a” should be “an”.
- Page 118, line 6:** One needs to write “in” between “dense” and “ $\Pi_2(X)$ ”.
- Page 118, Lemma 6.11, line 1:** “processes” should be “process”.
- Page 118, last line:** “alas” should be replaced by “that is”.
- Page 118, line 7:** “convergence” should be “converge”.
- Page 120, line 5 above Definition 6.13:** One needs a “.” after “Lemma 6.10”.
- Page 121, third line in the proof of Theorem 6.15:** Wrong accents on “càdlàg”.
- Page 121, line 9 after Theorem 6.15:** One could write that  $\sup_{t \geq 0} |(H_n \bullet X)_t - (H \bullet X)_t|$  converges in  $\mathcal{L}^2(\mathbb{P})$  instead.
- Page 121, fourth to last line in the proof of 6.15:** “ $H \bullet X_t(\omega)$ ” should be “ $(H \bullet X)_t(\omega)$ ”.
- Page 122, Theorem 6.16, line 1:** “bound” should be replaced with “bounded”.
- Page 122, Theorem 6.16, line 2:** “constant” should be replaced with “constants”.
- Page 123, the line after the second equation:** “Per” should be “By”.
- Page 123, the line after fifth equation:** “Schwartz” should be “Schwarz”.
- Page 124, Theorem 6.19:** We need to assume continuity of  $(X_t)$ .
- Page 125, line 3:** The reference should be to Exercise 5.7.
- Page 125:** Should be  $(X_t)$  instead of just  $X$ .
- Page 125, Lemma 6.20:** We must assume  $X$  to be continuous.
- Page 125, Exercise 6.10:** Enough to assume that  $\mathbb{E}[\langle X \rangle_t] < \infty$  for all  $t \geq 0$ .
- Page 129, Item (iv):** There’s infima missing in the definition of  $T_n$  and  $T'_n$ .
- Page 129 second line from the bottom:** “ $(H \bullet X_t)$ ” should be “ $(H \bullet X)_t$ ”.

**Page 130, proof of Theorem 7.4, definition of  $T_n$ :** The proper definition is

$$T_n = \inf \left\{ t \geq 0 \mid |X_t| \geq n \text{ or } \int_0^t H_s^2 d\langle X \rangle_s \geq n \text{ or } |X_t^T| \geq n \text{ or } \int_0^t H_s^2 d\langle X^T \rangle_s \geq n \right\}$$

**Page 134:** Write Exercise 8.4 as a theorem.

**Page 135, Lemma 7.11:** The process  $(Z_t)$  must be assumed progressively measurable.

**Page 135, line 8 from the top:** Remove the  $h$ , that is, replace  $\geq mh$  with  $m$ .

**Page 136:** Theorem 7.12 is not a true extension and should be removed.

**Page 136, Theorem 7.13:** One needs the word process to describe  $H$ .

**Page 137, line 8:** One need to add a subscript to  $H^{T_n}$ .

**Page 138, line 7 in Section 7.1:** The reference to Lemma 3.23 is wrong. It should be a reference to Lemma 3.29.

**Page 138, line 8 in Section 7.1:** “see” should be “seen”.

**Page 138, line 9 in Section 7.1:** Fundamental Theorem of Calculus is spelled with capital letters.

**Page 138, the proof of Main theorem 7.15:** The first paragraph is not needed. That is: we do not need the boundedness assumption.

**Page 139, in the third equation:** In the second sum one sums over “ $t_i^n \leq s$ ” instead of “ $t_i \leq s$ ”.

**Page 139, line 12** One should drop the word non-negative.

**Page 139, the line after the fourth equation:** One needs  $(X_s - X_{t_{k(s)}^n})^2$ .

**Page 139, sixth equation:**  $|X_s - X_{t_{k(s)}^n}|^2$ .

**Page 139–140:** One needs to write “ $c(X_{t_{i-1}^n}, X_{t_i^n})$ ” instead of “ $c(X_{t_i^n}, X_{t_{i-1}^n})$ ”.

**Page 140, first equation:** Write “[0, s]” instead of “ $(-\infty, s]$ ”.

**Page 140, second paragraph:** One needs to change “ $t_{i-1}^{n_k}$ ” into “ $t_{i-1}^{n_k}$ ”.

**Page 140, line 7:** One should interchange  $X_{t_{i-1}^{n_k}}$  and  $X_{t_i^{n_k}}$ .

**Page 140, line 7:** One should write “converge” instead of “convergence”.

**Page 142, line 5 after Theorem 8.3:** “Alas” should be changed to “therefore”.

**Page 143, Theorem 8.9:** Write “ $(H \cdot X)_t$ ” instead of “ $(\int_0^t H_s dX_s)_t$ ”.

**Page 144, Theorem 8.12:** One should write

$$\sup_{s \leq t} |Q_s^{\Delta_n}(X, Y) - \langle X, Y \rangle_s| \xrightarrow[n \rightarrow \infty]{\mathbb{P}} 0.$$

**Page 144, last line:** “Alas” should be changed to “therefore”.

**Page 146, fourth line from below:** Delete “the” before “extend”.

**Page 148, Theorem 8.21:** We must assume that the  $(X_t^i)$ 's are continuous.

**Page 151, seventh last line to third last line** Should be reformulated, in order it to make sense, replace the second “that” with “hence”.

**Page 153:** Figure 8.1 is wrong or a bit misleading. The distance between the graphs should be the same along the whole of the graphs. Note: This is not easy to fix in  $\LaTeX$ .

- Page 156, Exercise 8.3:** This is Exercise 7.2.
- Page 156, Exercise 8.4:** One must assume  $(X_t)$  is continuous.
- Page 156, Exercise 8.5:** " $\mathbf{1}_{(S,T]}$ " should be replaced by " $\mathbf{1}_{(S,T]}(s)$ ".
- Page 157, line 6 from top:** First arrow " $\rightarrow$ " should be Cartesian product  $\times$ .
- Page 157, item (iv) in Definition 9.2:** "then" should be "and".
- Page 157, first line below Definition 9.2:** Replace "explicitly" with "implicitly".
- Page 160, Equation (9.2):** Remove  $s$  after the inequality.
- Page 162, line 6 from top:** Missing parenthesis.
- Page 165, second line from top:** "Cauchy-Schwartz" should be "Cauchy-Schwarz".
- Page 169, line 9 from bottom:** Remove "then".
- Page 181, proof of Lemma 9.33 first line:** One need to add subscript  $t$  to  $\alpha$ .
- Page 181, proof of Lemma 9.33 part (ii):** The supremum does of course always exist.
- Page 182, proof of Lemma 9.34:** Change " $(\alpha_t Y_t)_t$ " to " $(\alpha_t Y_t)_{t \geq 0}$ ".
- Page 182 fourth line from the bottom** One need an "a" between "is" and "IP-martingale".
- Page 183, line two above Theorem 9.35:**  $\xi_t$  maps  $C$  into  $\mathbb{R}^d$ .
- Page 183, line 5 in Theorem 9.35:** Change "restrict" to "restriction".
- Page 183 proof of Lemma 9.36:** Delete the proof from "Let  $A \in \mathcal{F}_t$  then [...]". We are done at this point.
- Page 184, Theorem 9.37:** One must define  $A_t = (\alpha_t^{-1} \bullet \langle \alpha, X \rangle)_t$ .
- Page 184, First line in the proof of Theorem 9.37:** The line must end with "." instead of ",".
- Page 184, second equation:** Write " $\alpha^{-1} \alpha$ " instead of " $\alpha_s^{-1} \alpha_s$ ".
- Page 184, proof of Corollary 9.38:** In the first line one should add that  $(A_t)$  is continuous, adapted and of locally bounded variation.
- Page 184, Lemma 9.39, first line:** Delete "hereon" and write "this measurable space".
- Page 185, line 9 from bottom:** Replace "a  $N > 0$ " with "an  $N > 0$ ".
- Page 188, third line of Theorem 9.43:** One should note that  $(\mathcal{E}(X)_t)$  is continuous.
- Page 188, proof of Theorem 9.43:** The second part of the proof can be greatly simplified using Exercise 5.10. This implies that  $\mathbb{E}[Y_t^2] \leq 4M + 1$ .
- Page 188, Theorem 9.45:** "Levy" should be "Lévy".
- Page 188, Theorem 9.45:** The statement should be that  $(X_t)$  is a standard Brownian motion, and not any Brownian motion.
- Page 189 Theorem 9.46:** "Levy" should be "Lévy".
- Page 190, proof of Corollary 9.47:** "Levy" should be "Lévy" in the fourth line.
- Page 193, last line:** Integration is w.r.t.  $(B_t)$  not  $(X_t)$ .
- Page 194, Remark 9.50(C):** The integral in the first line should go to  $\infty$ .
- Page 194:** Theorem 9.51 should be Lemma 9.51 since it is not a central result.
- Page 194, third line from bottom:** "Cauchy-Schwartz" should be "Cauchy-Schwarz".

**Page 197, the line after the second equation:** One needs  $W \in \mathcal{L}^p(\mathbb{P}, \mathcal{B}_\infty)$  for some  $p > 1$ .

**Page 197:** The integrals in the fourth line are over  $\mathbb{R}^{nd}$ .

**Page 198, second line from the bottom:** One need to ad subscript “s” to  $H$ .

**Page 199, Theorem 9.54:** Wrong accents on “càdlàg”.

**Page 199, line 2 in Step 2:** Replace “a  $N \in \mathcal{B}_0$ ” with “an  $N \in \mathcal{B}_0$ ”. Beside the indicator function in “ $X_t \mathbf{1}_{N^c}$ ” should be bold:  $X_t \mathbf{1}_{N^c}$ .

**Page 202, Exercise 9.3, line 1:** “semiartingal” should be “semimartingale”.

**Page 202, Exercise 9.3, line 4:** One needs to add “the” between “is” and “set”.

**Page 202, Exercise 9.4 part (v):**  $\xi_t$  maps into  $\mathbb{R}^d$ .

**Page 202, second line from bottom:** “Cauchy-Schwartz” should be “Cauchy-Schwarz”.

**Page 206, fourth line from bottom:** “Cauchy-Schwartz” should be “Cauchy-Schwarz”.

**Page 207, Item (iv):**  $A^n$  should lie in  $\text{Mat}_{d,m}(\mathbb{Q})$ .

**Page 207, Definition 10.3:** The “ $C$ ” in the last equation should be “ $C_k$ ”.

**Page 208, Lemma 10.5:** Replace “ $\text{Mat}_{d,m}(\mathbb{R})(\mathbb{R})$ ” with “ $\text{Mat}_{d,m}(\mathbb{R})$ ”.

**Page 208, last paragraph:** We define the integral for every  $f : (a, b] \rightarrow \mathbb{R}^d$  which is measurable, and all the coordinate functions are integrable over  $(a, b]$ . Note that this is equivalent to  $f$  being measurable and

$$\int_a^b \|f(x)\| dx < \infty.$$

**Page 209, Lemma 10.6:** The statement of the lemma should be: Let  $f : (a, b] \rightarrow \mathbb{R}^d$  be measurable and assume that each coordinate function is integrable. Then

$$\left\| \int_a^b f(x) dx \right\| \leq \int_a^b \|f(x)\| dx.$$

**Page 209, fifth line from top:** “Cauchy-Schwartz” should be “Cauchy-Schwarz”.

**Page 210, Definition 10.7:** Large blank space in second line from the bottom should be ignored.

**Page 210, Example 10.8:** “ $x =$ ” should be removed from the first equation.

**Page 211, fourth equation** One needs a subscript “ $t$ ” on the variance process.

**Page 211, first equation above Lemma 10.11:** “ $X_t$ ” should be replaced with “ $X_0$ ”.

**Page 212, proof of Lemma 10.11, second line after the first equation:** “Schwartz” should be replaced by “Schwarz”.

**Page 212, proof of Lemma 10.11 in the second calculation:** The small  $t$  should be  $\tau$  in the last two lines.

**Page 212, proof of Lemma 10.11 after the second calculation:** We don’t use the stopping time lemma.

**Page 212, proof of Lemma 10.11 after third equation:** “local” should be deleted from the sentence.

**Page 212, proof of Lemma 10.11 fourth equation:** One needs a factor 4 after the inequality.



- Page 213, Theorem 10.12:** The Brownian motion should be  $m$ -dimensional.
- Page 213, line 8 in the proof of Theorem 10.12:** We conclude that that  $(X_t^{n+1})$  is adapted and not the integral.
- Page 213, line 11 in the proof of Theorem 10.12:** we have  $n \in \mathbb{N}$  and not  $n \in \mathbb{N}_0$ .
- Page 213, line 11 in the proof of Theorem 10.12:** Write " $K(\mathbb{R}^d)$ " instead of " $K(\mathbb{R})$ ".
- Page 213, fourth equation in the proof of Theorem 10.12:** In the end one should integrate over " $\Delta_n(s)$ " instead of " $\Delta_n(t)$ ".
- Page 213, proof of Theorem 10.12:** In the sixth line from the bottom to the third line from the bottom, replace " $\|\sigma(x)\|_{\text{op}}$ " with " $\|\sigma(x)\|_{\text{op}}^2$ ".
- Page 213, sixth equation in proof of Theorem 10.12:** One needs a " $C$ " outside the integral after second inequality.
- Page 214, second equation:** We have  $\|X_t^n - X_t^{n-1}\|^2$  instead of  $\|X_t^n - X_t^{n-1}\|$ .
- Page 214, sixth line:** Delete "of".
- Page 215, Definition 10.13, fourth line:** Replace "filter" with "filtration".
- Page 215, after Definition 10.13:** The integral is with respect to  $dB_s$ .
- Page 216, fifth line from bottom:** Replace "Gronwall's lemma" with "Grönwall's Lemma".
- Page 216, Lemma 10.16:** We need to assume that  $b, \sigma$  are Lipschitz on  $\overline{B(0, R)}$ .
- Page 216, second line from the bottom:** The observation that  $\phi(t) \leq 4R^2$  is to underline the fact that it is a map into  $[0, \infty)$ .
- Page 217, Theorem 10.17:** We need to assume that  $Y_0 = Z_0 = x_0$  for some  $x_0 \in \mathbb{R}^d$ .
- Page 217, Theorem 10.17, last line:** Write "solutions" instead of "solution".
- Page 217, Lemma 10.18:** We also have  $(X_{t \wedge T_n})$  is continuous.
- Page 218, proof of Lemma 10.18 around the middle:** One should write  $A_{m,n}^c \cap \{T_m > T_n\}$  instead of  $A_{m,n} \cap \{T_m > T_n\}$ .
- Page 219, second calculation:** Remove parenthesis around  $(B_s)$ , i.e., replace  $d(B_s)$  with  $dB_s$ .
- Page 219, Remark 10.19:** Note that if  $T$  is finite then  $\|X_t\|$  converges to infinity for  $t$  tending to  $T$ . Hence the name makes sense.
- Page 222, proof of Theorem 10.25 first line:** The sequence should converge to 0.
- Page 222, proof of Theorem 10.25, first equation:** The integral should be equal to  $n$ .
- Page 222, proof of Theorem 10.25, line after the third equation:** The support of  $\varphi_n$  is contained in  $(-a_n, a_n)^c$  and not  $(a_n, a_{n-1})$ .
- Page 223, second equation:** The integral is over  $\varphi_n'(y)$ .
- Page 223, fourth equation:**  $\Delta_t$  should be changed to  $\Delta_s$  in the two integrands where we integrate with respect to  $\Delta_s$ .
- Page 223, third line from the bottom:** We do not use the Stopping Time Lemma.
- Page 224, fourth equation:** The " $\leq$ " should be an " $=$ ".
- Page 224, proof of Theorem 10.25, third line up from the end of the proof:** "Gronwall" should be "Grönwall".

**Page 224, Remark 10.26(A):** The statement should be “A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is said to be **Hölder continuous of order  $\alpha \in (0, 1]$**  if there is a  $K > 0$  such that

$$|f(x) - f(y)| \leq K|x - y|^\alpha$$

for all  $x, y \in \mathbb{R}$ ”.

**Page 225, Example 10.28:** In the fourth line from the bottom one should write “Lévy” instead of “Levy”.

**Page 226, line 5-6:** Too many “such that”s, replace the second with a comma.

**Page 227, line 10 from top:** “an intersection stable” *not* “a intersection stable”.

**Page 230, Lemma 10.32, second line:** Replace “filter” with “filtration” and put parentheses around  $\overline{\mathcal{F}}_t$ .

**Page 232, proof of Theorem 10.35(ii), first line:**  $MP_x$  should not be in italics, but instead upright:  $MP_x$ .

**Page 232, fourth line after the proof of Theorem 10.35(ii):** One should include the filtration in the measurable space  $(\Omega, \mathcal{F})$ . That is one should write “ $(\Omega, \mathcal{F}, (\mathcal{F}_t))$ ”.

**Page 233, first line of Remark 10.37:** Here we take  $Y = f(X_t)$ .

**Page 233, Theorem 10.39:** It should be added that the process is a strong Markov process under  $(\mathbb{P}_x)_{x \in \mathbb{R}^d}$ .

**Page 234, thirteenth line from top:** “Cauchy-Schwartz” should be “Cauchy-Schwarz”.

**Page 235, proof of Theorem 10.39:** In the italics it should be noted that the proof is for  $j \neq 0$  only.

**Page 236, Corollary 10.40:** The equality should read

$$\mathbb{E}[f(\overline{X}_{s+t}) \mid \mathcal{F}_s^{\overline{X}}] = T_t(f)(\overline{X}_s).$$

**Page 236, Corollary 10.40:** One needs to assume that  $f$  is measurable.

**Page 237, Theorem 10.41:** In the first line  $\sigma$  should map  $\mathbb{R}^d$  into  $\text{Mat}_{d,d}(\mathbb{R})$ , and the other “ $\text{Mat}_{d,d}(\mathbb{R})(\mathbb{R})$ ” should also be replaced by  $\text{Mat}_{d,d}(\mathbb{R})$ .

**Page 237, Theorem 10.41(iv):** Replace “where  $(Z_t)$  is a any [...]” with “where  $(Z_t)$  is any [...]”.

**Page 237, proof of Theorem 10.41:** Second line in second paragraph replace “ $(\overline{\mathcal{F}}_t)$ -martingale” with “ $(\overline{\mathcal{F}}_t)$ -martingales”.

**Page 237, Proof of Theorem 10.41, second equation:** One needs to add superscript “ $i$ ” to  $X_{t \wedge T}$  in both lines. Furthermore in the first line we need a subscript “ $s$ ” on  $X$  in the first integral and a “ $ds$ ” in end of the second integral.

**Page 237, Proof of Theorem 10.41, fourth line from the bottom:** What is really proven, is that  $(X_{t \wedge T})$  is a semimartingale with local variation part

$$\int_0^{t \wedge T} (\beta + b)(X_s) ds.$$

**Page 238, Theorem 10.43(ii):** We need to assume that both  $b$  and  $\beta$  are locally bounded.

**Page 238, proof of Theorem 10.43(ii):** Replace “assume” with “Assume” in the first line.

**Page 238, proof of Theorem 10.43(ii):** In the line above first equation replace  $Y$  by  $Z$ .

**Page 238, proof of Theorem 10.43(ii):** Replace  $Y$  with  $Z$  in the first equation.

**Page 240, Theorem 10.44 first line in the  $\Leftarrow$  part:** Replace  $MP_x(b, a)$  with  $MP_x(b + \beta, a)$ .

**Page 240, Corollary 10.46:** In assumption (iii) one needs

$$\|\sigma(x)\| \leq A(1 + \|x\|).$$

**Page 242, Exercise 10.3:** “Gronwall” should be changed to “Grönwall”.

**Page 248, eleventh line from bottom:** “Cauchy-Schwartz” should be “Cauchy-Schwarz”.

**Page 249, line 2 from top:** Replace “there exists a  $N$ ” with “there exists an  $N$ ”.

**Page 249, fifth line from top:** Replace whole line with “By help of the  $(\sigma)$ -additivity of measures, Cauchy-Schwarz’ inequality implies that”.

**Page 252, third line below Definition A.7:** Two errors, replace “Remember that [...] functions in  $\mathcal{R}$   $\mathcal{R}$ - $\mathcal{B}(\mathbb{R})$ -measurable.” with “Remember that [...] functions in  $\mathcal{R}$   $\sigma(\mathcal{R})$ - $\mathcal{B}(\mathbb{R})$ -measurable.”

**Page 261, Lemma A.16, second line:** Replace “sucg” and “measureable” with “such” and “measurable”, respectively.

**Page 261, fourth line from bottom:** Ugly formulation.

**Page 261, third line from bottom:** polish should be with capital P.

**Page 262, proof of Lemma A.26:** The last four  $P$ ’s are wrong, they should be  $\mathbb{P}$ .

**Page 263, Lemma A.21** Weiner measure should be bold: **Weiner measure**.

**Page 264:** Lemma A.22 is not needed, so it should be overlooked.

**Page 268, Theorem A.25, first line:** “ $\text{Mat}_{m,d}(\mathbb{R})(\mathbb{R})$ ” should be “ $\text{Mat}_{m,d}(\mathbb{R})$ ”.

**Page 268, Theorem A.25, first item:** Replace “filter” with “filtration”.

**Page 269, Definition A.26:** polish should be with capital P.

**Page 270:** polish should be with capital P.

**Page 275, Theorem A.33:** The range of the maps should be contained in  $\mathbb{R}$  and they should be  $\mathcal{B}(S)$ - $\mathcal{B}(\mathbb{R})$ -measurable.

**Page 275, Theorem A.33:** Change “a  $s \in S$ ” with “an  $s \in S$ ”.

**Page 275, line 7 from the bottom:** Replace “Choose now according Skorokhod’s” with “Choose now according to Skorokhod’s”.

**Page 279:** Line 7 from top should be reformulated.

**Page 285, first line on the page:** There is an “a” too much. I.e. “Let  $f : \mathbb{R}^m \rightarrow V$  be a locally Lipschitz” should be replaced by: “Let  $f : \mathbb{R}^m \rightarrow V$  be locally Lipschitz”.

**Page 286, Theorem B.8:** Too many “we find that”.

**Page 289, Definition C.1:** Replace “The pair  $(S, d)$  is the called a **metric space**.” with “The pair  $(S, d)$  is called a **metric space**.” Moreover replace period in (v) with comma and “The” with “then” below.

**Page 299, proof of Lemma C.10:** The supremum is over  $y$  not  $x$ .

**Page 300, Theorem C.11:** Replace “Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  convex [...]” with “Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be convex [...]”.

**Page 304:** Stochastic is spelled incorrectly.

**Page 305:** Replace “Gronwall’s inequality, 242” with “Grönwall’s inequality, 242”.

**Page 306:** Two of the Lévy’s Theorem entries should be put together, they were not because of incorrect spelling of Lévy.